

Correction de la planche d'exercices 1

Exercice 1

- a. Les entiers naturels sont 0 et $\sqrt{25} = 5$.
 b. Les entiers relatifs sont 0, -1 , $-\frac{8}{4} = -2$ et $\sqrt{25} = 5$.
 c. Les rationnels sont $0 = \frac{0}{3}$, $-1 = \frac{-1}{1}$, $\frac{4}{3}$, $-\frac{8}{4} = \frac{-8}{4}$, $2,5 = \frac{5}{2}$ et $\sqrt{25} = \frac{5}{1}$.
 d. Tous les nombres donnés sont des réels.

Exercice 2 La proposition est fausse. En effet, pour $x = -3$, on a $-x = 3$ et 3 est positif.

Exercice 3

- a. Vrai. b. Vrai. c. Vrai. d. Vrai.

Exercice 4

a.

$$\begin{aligned} 67 + 94 + 33 &= (67 + 94) + 33 \\ &= 67 + (94 + 33) \\ &= 67 + (33 + 94) \\ &= (67 + 33) + 94 \\ &= 100 + 94 \\ &= 194. \end{aligned}$$

b. En procédant comme à la question précédente, on obtient

$$4 \times 39 \times 25 = 4 \times 25 \times 39 = 100 \times 39 = 3900.$$

c.

$$-1,2 - 5 + 0,2 = (-1,2) + (-5) + 0,2 = (-1,2) + 0,2 + (-5) = (-1) + (-5) = -6.$$

d.

$$-0,25 \times 49 \times (-8) = (-0,25) \times 49 \times (-8) = (-0,25) \times (-8) \times 49 = 2 \times 49 = 98.$$

Exercice 5

- a. Faux. b. Vrai. c. Faux. d. Vrai.

Exercice 6

a.

$$\frac{6}{8} = \frac{3 \times \cancel{2}}{4 \times \cancel{2}} = \frac{3}{4}.$$

b.

$$\frac{10}{15} = \frac{2 \times \cancel{5}}{3 \times \cancel{5}} = \frac{2}{3}.$$

c.

$$\frac{18}{12} = \frac{3 \times \cancel{6}}{2 \times \cancel{6}} = \frac{3}{2}.$$

d.

$$\frac{45}{60} = \frac{3 \times \cancel{15}}{4 \times \cancel{15}} = \frac{3}{4}.$$

e.

$$\frac{90}{72} = \frac{45 \times \cancel{2}}{36 \times \cancel{2}} = \frac{45}{36} = \frac{5 \times \cancel{9}}{4 \times \cancel{9}} = \frac{5}{4}.$$

f.

$$\frac{105}{140} = \frac{21 \times \cancel{5}}{28 \times \cancel{5}} = \frac{21}{28} = \frac{3 \times \cancel{7}}{4 \times \cancel{7}} = \frac{3}{4}.$$

Exercice 7

a.

$$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}.$$

b.

$$\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}.$$

c.

$$\frac{5}{6} + \frac{2}{9} = \frac{15}{18} + \frac{4}{18} = \frac{19}{18}.$$

d.

$$\frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12}.$$

e.

$$2 + \frac{1}{3} = \frac{2}{1} + \frac{1}{3} = \frac{6}{3} + \frac{1}{3} = \frac{7}{3}.$$

f.

$$\frac{3}{4} + 3 = \frac{3}{4} + \frac{3}{1} = \frac{3}{4} + \frac{12}{4} = \frac{15}{4}.$$

Exercice 8

a.

$$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}.$$

b.

$$\frac{5}{6} \times \frac{2}{9} = \frac{5 \times 2}{6 \times 9} = \frac{5 \times \cancel{2}}{\cancel{2} \times 3 \times 9} = \frac{5}{27}.$$

c.

$$\frac{7}{12} \times \frac{18}{21} = \frac{7 \times 18}{12 \times 21} = \frac{\cancel{7} \times \cancel{3} \times \cancel{6}}{\cancel{6} \times 2 \times \cancel{3} \times \cancel{7}} = \frac{1}{2}.$$

d.

$$\frac{3}{8} \times \frac{8}{3} = \frac{\cancel{3} \times \cancel{8}}{\cancel{8} \times \cancel{3}} = 1.$$

e.

$$3 \times \frac{2}{5} = \frac{3 \times 2}{5} = \frac{6}{5}.$$

f.

$$\frac{3}{4} \times 8 = \frac{3 \times 8}{4} = \frac{24}{4} = 6.$$

Exercice 9 Voici les paires de nombres opposés : $\frac{5}{2}$ et $-\frac{5}{2}$, $\frac{2}{5}$ et $\frac{2}{-5}$, $\frac{3}{4}$ et $\frac{-3}{4}$, $\frac{-4}{3}$ et $\frac{4}{3}$.

Exercice 10

- a. L'inverse de $\frac{7}{5}$ est $\frac{5}{7}$.
 b. L'inverse de $-\frac{4}{9}$ est $-\frac{9}{4}$.
 c. L'inverse de $\frac{1}{6}$ est 6.
 d. L'inverse de -6 est $-\frac{1}{6}$.

Exercice 11

- a. $\frac{3}{5} \times \frac{5}{3} = 1$ b. $-\frac{1}{4} \times (-4) = 1$ c. $8 \times \frac{1}{8} = 1$

Exercice 12

a.

$$\frac{3}{5} : \frac{2}{7} = \frac{3}{5} \times \frac{7}{2} = \frac{3 \times 7}{5 \times 2} = \frac{21}{10}.$$

b.

$$\frac{2}{3} : \frac{5}{6} = \frac{2}{3} \times \frac{6}{5} = \frac{2 \times 6}{3 \times 5} = \frac{2 \times \cancel{3} \times 2}{\cancel{3} \times 5} = \frac{4}{5}.$$

c.

$$\frac{9}{7} : \frac{15}{14} = \frac{9}{7} \times \frac{14}{15} = \frac{9 \times 14}{7 \times 15} = \frac{\cancel{3} \times 3 \times \cancel{7} \times 2}{\cancel{7} \times \cancel{3} \times 5} = \frac{6}{5}.$$

d.

$$\frac{3}{4} : 2 = \frac{3}{4} \times \frac{1}{2} = \frac{3 \times 1}{4 \times 2} = \frac{3}{8}.$$

e.

$$\frac{8}{7} : 4 = \frac{8}{7} \times \frac{1}{4} = \frac{\cancel{4} \times 2 \times 1}{7 \times \cancel{4}} = \frac{2}{7}.$$

f.

$$5 : \frac{5}{6} = \cancel{5} \times \frac{6}{\cancel{5}} = 6.$$

Exercice 13

a.

$$7 \times \left(-\frac{2}{7}\right) + \frac{3}{4} = -2 + \frac{3}{4} = -\frac{8}{4} + \frac{3}{4} = -\frac{5}{4}.$$

b.

$$\frac{4}{5} - \frac{3}{25} \times \frac{5}{6} = \frac{4}{5} - \frac{3 \times 5}{25 \times 6} = \frac{4}{5} - \frac{\cancel{3} \times \cancel{5}}{\cancel{5} \times 5 \times \cancel{3} \times 2} = \frac{4}{5} - \frac{1}{10} = \frac{8}{10} - \frac{1}{10} = \frac{7}{10}.$$

c.

$$27 : \left(\frac{1}{2} - \frac{7}{12}\right) = 27 : \left(\frac{6}{12} - \frac{7}{12}\right) = 27 : \left(-\frac{1}{12}\right) = 27 \times (-12) = -324.$$

d.

$$\frac{\frac{3}{5} - \frac{1}{4}}{1 + \frac{2}{5}} = \frac{\frac{12}{20} - \frac{5}{20}}{\frac{7}{5}} = \frac{\frac{7}{20}}{\frac{7}{5}} = \frac{\cancel{7}}{20} \times \frac{5}{\cancel{7}} = \frac{1}{4}.$$

e.

$$1 + \frac{2}{5} - \frac{1}{14} \times \frac{28}{5} = 1 + \frac{2}{5} - \frac{1 \times 28}{14 \times 5} = 1 + \frac{2}{5} - \frac{1 \times \cancel{14} \times 2}{\cancel{14} \times 5} = 1 + \frac{2}{5} - \frac{2}{5} = 1.$$

f.

$$\frac{1 - \frac{3}{8} - \frac{5}{12}}{1 + \frac{3}{4} - \frac{5}{12}} = \frac{\frac{24}{24} - \frac{9}{24} - \frac{10}{24}}{\frac{12}{12} + \frac{9}{12} - \frac{5}{12}} = \frac{\frac{5}{24}}{\frac{16}{12}} = \frac{\frac{5}{24}}{\frac{4}{3}} = \frac{5}{24} \times \frac{3}{4} = \frac{5 \times 3}{24 \times 4} = \frac{5 \times \cancel{3}}{\cancel{3} \times 8 \times 4} = \frac{5}{32}.$$

Exercice 14

- a. $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32.$
b. $5^2 = 5 \times 5 = 25.$
c. $(-2)^5 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) = -32.$
d. $(-5)^2 = (-5) \times (-5) = 25.$
e. $-2^5 = -(2^5) = -32.$
f. $-5^2 = -(5^2) = -25.$

Exercice 15

- a. $2^0 = 1, 2^{-1} = \frac{1}{2}, 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$ et $2^{-3} = \frac{1}{2^3} = \frac{1}{8}.$
b. $3^0 = 1, 3^{-1} = \frac{1}{3}, 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ et $3^{-3} = \frac{1}{3^3} = \frac{1}{27}.$
c. $10^0 = 1, 10^{-1} = \frac{1}{10}, 10^{-2} = \frac{1}{10^2} = \frac{1}{100}$ et $10^{-3} = \frac{1}{10^3} = \frac{1}{1000}.$

Exercice 16

- a. $2^4 \times 2^7 = 2^{11}.$ b. $3^{-2} \times 3^6 = 3^4.$ c. $5^{-3} \times 5^{-4} = 5^{-7}.$ d. $7^8 \times 7^{-5} = 7^3.$

Exercice 17

- a. $\frac{2^9}{2^4} = 2^5.$ b. $\frac{3^{-1}}{3^4} = 3^{-5}.$ c. $\frac{5^2}{5^{-3}} = 5^5.$ d. $\frac{7^{-2}}{7^{-6}} = 7^4.$

Exercice 18

- a. $(2^3)^4 = 2^{12}.$ b. $(3^4)^{-2} = 3^{-8}.$ c. $(5^{-1})^{-7} = 5^7.$ d. $(7^{-5})^3 = 7^{-15}.$

Exercice 19

- a. $(2 \times 3)^4 = 2^4 \times 3^4.$ b. $(5 \times 2)^3 = 5^3 \times 2^3.$ c. $(7 \times 3)^5 = 7^5 \times 3^5.$

Exercice 20

- a. $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4}.$ b. $\left(\frac{7}{5}\right)^6 = \frac{7^6}{5^6}.$ c. $\left(\frac{5}{2}\right)^3 = \frac{5^3}{2^3}.$

Exercice 21

- a. $3^8 \times 3^{-2} = 3^6.$
b. $\frac{5^{-5}}{5^2} = 5^{-7}.$
c. $(7^{-4})^{-1} = 7^4.$
d. $11^{-5} \times 11^{-6} = 11^{-11}.$
e. $\frac{2^2}{2^{-3}} = 2^5.$
f. $(3^{-4})^4 = 3^{-16}.$

Exercice 22**a.**

$$2 \times 2^6 = 2^7.$$

b.

$$\frac{5^4}{5} = \frac{5^4}{5^1} = 5^3.$$

c.

$$(5^3)^2 \times 5^{-4} = 5^6 \times 5^{-4} = 5^2.$$

d.

$$\frac{3^7 \times 3^{-2}}{3^3} = \frac{3^5}{3^3} = 3^2.$$

e.

$$7^{-1} \times 7^2 \times 7^{-4} = 7^1 \times 7^{-4} = 7^{-3}.$$

f.

$$2^5 \times 3^5 \times 6^{-3} = (2 \times 3)^5 \times 6^{-3} = 6^5 \times 6^{-3} = 6^2.$$

g.

$$\frac{2^4 \times 5^4}{10^3} = \frac{(2 \times 5)^4}{10^3} = \frac{10^4}{10^3} = 10^1 = 10.$$

h.

$$16^{-1} \times 2^6 = (2^4)^{-1} \times 2^6 = 2^{-4} \times 2^6 = 2^2 = 4.$$

i.

$$\frac{25^2}{5^{-1} \times 125} = \frac{(5^2)^2}{5^{-1} \times 5^3} = \frac{5^4}{5^2} = 5^2 = 25.$$

Exercice 23 On a

$$4^7 = (2^2)^7 = 2^{14}.$$

Or $2^{14} < 2^{15}$, donc

$$4^7 < 2^{15}.$$

Exercice 24**a.** Faux : la moitié de 2^6 est 2^5 .**b.** Vrai.**c.** Faux : le double de 6^4 est 2×6^4 , et non $6^5 = 6 \times 6^4$.**d.** Faux : l'opposé de 5^2 est $-5^2 = -25$, tandis que $5^{-2} = \frac{1}{25}$.**e.** Faux : le triple de 4^2 est $3 \times 4^2 = 3 \times 16 = 48$, tandis que $12^2 = 144$.**Exercice 25**

a. $(-4)^3 = -64.$

b. $4^{-3} = \frac{1}{64}.$

c. $(-4)^{-3} = -\frac{1}{64}.$

Exercice 26**a.** Vrai : $-\sqrt{2}$ est l'opposé de $\sqrt{2}$.**b.** Faux : la racine carrée d'un réel strictement négatif n'existe pas.**c.** Vrai : $3^2 = (-3)^2 = 9$.**d.** Vrai : $(-3)^2 = 9$.**e.** Faux : une racine carrée est toujours positive.**f.** Vrai : $\sqrt{9} = 3$.

Exercice 27

- a. $\sqrt{49} = 7$. b. $\sqrt{64} = 8$. c. $\sqrt{1} = 1$. d. $\sqrt{0} = 0$. e. $\sqrt{0,81} = 0,9$. f. $\sqrt{0,01} = 0,1$.

Exercice 28

- a. On a $\sqrt{16+9} = \sqrt{25} = 5$ et $\sqrt{16} + \sqrt{9} = 4 + 3 = 7$, donc ces nombres ne sont pas égaux.
 b. On a $\sqrt{25-16} = \sqrt{9} = 3$ et $\sqrt{25} - \sqrt{16} = 5 - 4 = 1$, donc ces nombres ne sont pas égaux.

Exercice 29

a.

$$(\sqrt{5})^2 = 5.$$

b.

$$\sqrt{6^2} = 6.$$

c.

$$\sqrt{(-7)^2} = 7.$$

d.

$$\sqrt{3} \times \sqrt{3} = (\sqrt{3})^2 = 3.$$

e.

$$\sqrt{2} \times 3 \times \sqrt{2} = 3 \times \sqrt{2} \times \sqrt{2} = 3 \times (\sqrt{2})^2 = 6.$$

f.

$$\sqrt{5} \times 2\sqrt{5} = \sqrt{5} \times 2 \times \sqrt{5} = 2 \times \sqrt{5} \times \sqrt{5} = 2 \times (\sqrt{5})^2 = 10.$$

g.

$$3\sqrt{7} \times 2 \times \sqrt{7} = 3 \times 2 \times \sqrt{7} \times \sqrt{7} = 6 \times 7 = 42.$$

h.

$$4\sqrt{5} \times 2\sqrt{5} = 4 \times \sqrt{5} \times 2 \times \sqrt{5} = 4 \times 2 \times \sqrt{5} \times \sqrt{5} = 8 \times 5 = 40.$$

i.

$$(2\sqrt{3})^2 = 2^2 \times (\sqrt{3})^2 = 4 \times 3 = 12.$$

j.

$$(-3\sqrt{6})^2 = (-3)^2 \times (\sqrt{6})^2 = 9 \times 6 = 54.$$

k.

$$(\sqrt{2})^3 = (\sqrt{2})^2 \times \sqrt{2} = 2 \times \sqrt{2} = 2\sqrt{2}.$$

l.

$$(\sqrt{2})^4 = [(\sqrt{2})^2]^2 = 2^2 = 4.$$

Exercice 30

- a. $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$.
 b. $\sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$.
 c. $\sqrt{75} = \sqrt{25 \times 3} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$.
 d. $\sqrt{180} = \sqrt{36 \times 5} = \sqrt{36} \times \sqrt{5} = 6\sqrt{5}$.

Exercice 31

- a. $\sqrt{\frac{64}{9}} = \frac{\sqrt{64}}{\sqrt{9}} = \frac{8}{3}$.
 b. $\sqrt{\frac{1}{36}} = \frac{\sqrt{1}}{\sqrt{36}} = \frac{1}{6}$.

c. $\sqrt{\frac{3}{25}} = \frac{\sqrt{3}}{\sqrt{25}} = \frac{\sqrt{3}}{5}.$

d. $\sqrt{0,75} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}.$

Exercice 32

a.

$$\sqrt{5} + \sqrt{20} = \sqrt{5} + 2\sqrt{5} = 3\sqrt{5}.$$

b.

$$\sqrt{18} - \sqrt{2} = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}.$$

c.

$$\sqrt{27} - \sqrt{12} + \sqrt{3} = 3\sqrt{3} - 2\sqrt{3} + \sqrt{3} = 2\sqrt{3}.$$

d.

$$-\sqrt{700} - \sqrt{63} = -10\sqrt{7} - 3\sqrt{7} = -13\sqrt{7}.$$

e.

$$2\sqrt{48} - 3\sqrt{75} + \sqrt{300} = 2(4\sqrt{3}) - 3(5\sqrt{3}) + 10\sqrt{3} = 8\sqrt{3} - 15\sqrt{3} + 10\sqrt{3} = 3\sqrt{3}.$$

f.

$$\frac{\sqrt{200} + \sqrt{50}}{\sqrt{2}} = \frac{10\sqrt{2} + 5\sqrt{2}}{\sqrt{2}} = \frac{15\sqrt{2}}{\sqrt{2}} = 15.$$

Exercice 33

a. $\frac{1}{\sqrt{3}} = \frac{1 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{\sqrt{3}}{3}.$

b. $\frac{2}{\sqrt{5}} = \frac{2 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{2\sqrt{5}}{5}.$

c. $\frac{7}{\sqrt{7}} = \frac{7 \times \sqrt{7}}{\sqrt{7} \times \sqrt{7}} = \sqrt{7}.$

Exercice 34

a. $|2| = 2.$

b. $|-4| = 4.$

c. $|0| = 0.$

d. $|1 + (-2)| = |-1| = 1.$

e. Comme $1 - \sqrt{2} < 0$, on obtient $|1 - \sqrt{2}| = -(1 - \sqrt{2}) = \sqrt{2} - 1.$

f. $|(-3) \times (-5)| = |-3| \times |-5| = 3 \times 5 = 15.$

g. $\left|\frac{2}{-7}\right| = \frac{|2|}{|-7|} = \frac{2}{7}.$

h. Comme $10^{-2} \geq 0$, on obtient $|10^{-2}| = 10^{-2} = \frac{1}{100}.$

Exercice 35 La troisième proposition est vraie. En effet, pour tout réel x , on a

$$\sqrt{4x^2} = \sqrt{4 \times x^2} = \sqrt{4} \times \sqrt{x^2} = 2|x|.$$