

Correction de la planche d'exercices 2

Exercice 1

- a. $3(x + 5) = 3x + 15.$
- b. $-4(x - 2) = -4x + 8.$
- c. $(2x - 1) \times 5 = 10x - 5.$
- d. $(-6x + 2) \times (-3) = 18x - 6.$
- e. $\frac{3}{2}(x - 4) = \frac{3}{2}x - 6.$
- f. $-\frac{2}{3}(3x + 6) = -2x - 4.$
- g. $\sqrt{2}(x - \sqrt{3}) = \sqrt{2}x - \sqrt{6}.$
- h. $-\sqrt{3}(\sqrt{3}x - 1) = -3x + \sqrt{3}.$
- i. $5 - 2(x + 3) = 5 - 2x - 6 = -2x - 1.$
- j. $7x - 3(-x + 4) = 7x + 3x - 12 = 10x - 12.$
- k. $2(x - 3) + x(x + 1) = 2x - 6 + x^2 + x = x^2 + 3x - 6.$
- l. $4(2x - 1) - 3(x + 5) = 8x - 4 - 3x - 15 = 5x - 19.$

Exercice 2

- a. $(x + 4)(x + 1) = x^2 + x + 4x + 4 = x^2 + 5x + 4.$
- b. $(x - 3)(x + 5) = x^2 + 5x - 3x - 15 = x^2 + 2x - 15.$
- c. $(2x + 3)(x - 4) = 2x^2 - 8x + 3x - 12 = 2x^2 - 5x - 12.$
- d. $(3x - 2)(-2x + 1) = -6x^2 + 3x + 4x - 2 = -6x^2 + 7x - 2.$
- e. $(5 - x)(2x - 3) = 10x - 15 - 2x^2 + 3x = -2x^2 + 13x - 15.$
- f. $(2x - 1)(3 - x) = 6x - 2x^2 - 3 + x = -2x^2 + 7x - 3.$
- g. $\left(x + \frac{1}{2}\right)(x - 3) = x^2 - 3x + \frac{1}{2}x - \frac{3}{2} = x^2 - \frac{5}{2}x - \frac{3}{2}.$
- h. $(x + 6)\left(2x - \frac{1}{3}\right) = 2x^2 - \frac{1}{3}x + 12x - 2 = 2x^2 + \frac{35}{3}x - 2.$
- i. $(x - \sqrt{2})(x - \sqrt{3}) = x^2 - \sqrt{3}x - \sqrt{2}x + \sqrt{6} = x^2 - (\sqrt{2} + \sqrt{3})x + \sqrt{6}.$
- j. $(\sqrt{5}x + 1)(x - \sqrt{5}) = \sqrt{5}x^2 - 5x + x - \sqrt{5} = \sqrt{5}x^2 - 4x - \sqrt{5}.$
- k. $(x + 2)(x^2 - 3x + 1) = x^3 - 3x^2 + x + 2x^2 - 6x + 2 = x^3 - x^2 - 5x + 2.$
- l. $(x^2 + 3x - 2)(-3x + 2) = -3x^3 + 2x^2 - 9x^2 + 6x + 6x - 4 = -3x^3 - 7x^2 + 12x - 4.$

Exercice 3

- a. Faux.
- b. Vrai.
- c. Vrai.

Exercice 4

a.

$$\begin{aligned} 2(x + 1)(x - 3) &= 2(x^2 - 3x + x - 3) \\ &= 2(x^2 - 2x - 3) \\ &= 2x^2 - 4x - 6. \end{aligned}$$

b.

$$\begin{aligned} (x + 4)(x - 2)(x - 1) &= (x^2 - 2x + 4x - 8)(x - 1) \\ &= (x^2 + 2x - 8)(x - 1) \\ &= x^3 - x^2 + 2x^2 - 2x - 8x + 8 \\ &= x^3 + x^2 - 10x + 8. \end{aligned}$$

c.

$$\begin{aligned}
 3(x-2)(x+4) - x(x+2) &= 3(x^2 + 4x - 2x - 8) - x^2 - 2x \\
 &= 3(x^2 + 2x - 8) - x^2 - 2x \\
 &= 3x^2 + 6x - 24 - x^2 - 2x \\
 &= 2x^2 + 4x - 24.
 \end{aligned}$$

d.

$$\begin{aligned}
 (x+1)(1-2x)(-3x+1) &= (x-2x^2+1-2x)(-3x+1) \\
 &= (-2x^2-x+1)(-3x+1) \\
 &= 6x^3-2x^2+3x^2-x-3x+1 \\
 &= 6x^3+x^2-4x+1.
 \end{aligned}$$

Exercice 5

- a. $6x + 18 = 6(x + 3)$.
- b. $15x - 10 = 5(3x - 2)$.
- c. $x^2 + 7x = x(x + 7)$.
- d. $2x^3 - x^2 = x^2(2x - 1)$.
- e. $4x^2 + 8x = 4x(x + 2)$.
- f. $x^4 - 3x^2 = x^2(x^2 - 3)$.
- g. $x^3 + x^2 + x = x(x^2 + x + 1)$.
- h. $x^4 + 2x^3 - 3x^2 = x^2(x^2 + 2x - 3)$.

Exercice 6

a.

$$5x(x-2) + 3(x-2) = (5x+3)(x-2).$$

b.

$$x(3x+1) - 2(3x+1) = (x-2)(3x+1).$$

c.

$$\begin{aligned}
 (x+4)(2x-1) + 3(x+4) &= (x+4)[(2x-1)+3] \\
 &= (x+4)[2x-1+3] \\
 &= (x+4)(2x+2) \\
 &= 2(x+4)(x+1).
 \end{aligned}$$

d.

$$\begin{aligned}
 (x+5)(5x-3) - (x+5)(2x+1) &= (x+5)[(5x-3)-(2x+1)] \\
 &= (x+5)[5x-3-2x-1] \\
 &= (x+5)(3x-4).
 \end{aligned}$$

e.

$$\begin{aligned}
 (2x+1)^2 + 4(2x+1) &= (2x+1)(2x+1) + 4(2x+1) \\
 &= (2x+1)[(2x+1)+4] \\
 &= (2x+1)[2x+1+4] \\
 &= (2x+1)(2x+5).
 \end{aligned}$$

f.

$$\begin{aligned}
 (3x-2)^2 - (x+1)(3x-2) &= (3x-2)(3x-2) - (x+1)(3x-2) \\
 &= (3x-2)[(3x-2)-(x+1)] \\
 &= (3x-2)[3x-2-x-1] \\
 &= (3x-2)(2x-3).
 \end{aligned}$$

g.

$$\begin{aligned}
 (4x-1)(x+1) + 3(x+1)(x+2) &= (x+1)[(4x-1) + 3(x+2)] \\
 &= (x+1)[4x-1+3x+6] \\
 &= (x+1)(7x+5).
 \end{aligned}$$

h.

$$\begin{aligned}
 (x-4)(6x+9) - (2x+3)(x+1) &= 3(x-4)(2x+3) - (2x+3)(x+1) \\
 &= (2x+3)[3(x-4) - (x+1)] \\
 &= (2x+3)[3x-12-x-1] \\
 &= (2x+3)(2x-13).
 \end{aligned}$$

Exercice 7**a.**

$$\begin{aligned}
 (x-1)^2 + 2(x-1) + (2x+5)(x-1) &= (x-1)(x-1) + 2(x-1) + (2x+5)(x-1) \\
 &= (x-1)[(x-1) + 2 + (2x+5)] \\
 &= (x-1)[x-1+2+2x+5] \\
 &= (x-1)(3x+6) \\
 &= 3(x-1)(x+2).
 \end{aligned}$$

b.

$$\begin{aligned}
 (x+2)(x-3) + (x+2)^2 - 4(x+2) &= (x+2)(x-3) + (x+2)(x+2) - 4(x+2) \\
 &= (x+2)[(x-3) + (x+2) - 4] \\
 &= (x+2)[x-3+x+2-4] \\
 &= (x+2)(2x-5).
 \end{aligned}$$

c.

$$\begin{aligned}
 (2x-1)(x+4) - (2x-1)(x-3) + (2x-1)^2 &= (2x-1)(x+4) - (2x-1)(x-3) \\
 &\quad + (2x-1)(2x-1) \\
 &= (2x-1)[(x+4) - (x-3) + (2x-1)] \\
 &= (2x-1)[x+4-x+3+2x-1] \\
 &= (2x-1)(2x+6) \\
 &= 2(2x-1)(x+3).
 \end{aligned}$$

d.

$$\begin{aligned}
 (x-5)(3x+1) - 2(x-5) + (x-5)^2 &= (x-5)(3x+1) - 2(x-5) + (x-5)(x-5) \\
 &= (x-5)[(3x+1) - 2 + (x-5)] \\
 &= (x-5)[3x+1-2+x-5] \\
 &= (x-5)(4x-6) \\
 &= 2(x-5)(2x-3).
 \end{aligned}$$

e.

$$\begin{aligned}
 (3x+1)(x-1) + (6x+2)(2x+5) - (3x+1)^2 &= (3x+1)(x-1) + 2(3x+1)(2x+5) \\
 &\quad - (3x+1)(3x+1) \\
 &= (3x+1)[(x-1) + 2(2x+5) - (3x+1)] \\
 &= (3x+1)[x-1+4x+10-3x-1]
 \end{aligned}$$

$$\begin{aligned}
 &= (3x + 1)(2x + 8) \\
 &= 2(3x + 1)(x + 4).
 \end{aligned}$$

f.

$$\begin{aligned}
 (x + 1)(x - 2) + (x + 1)(3x + 4) + 5x + 5 &= (x + 1)(x - 2) + (x + 1)(3x + 4) + 5(x + 1) \\
 &= (x + 1)[(x - 2) + (3x + 4) + 5] \\
 &= (x + 1)[x - 2 + 3x + 4 + 5] \\
 &= (x + 1)(4x + 7).
 \end{aligned}$$

Exercice 8 La proposition est fausse. En effet, pour $x = 1$, on a

$$(x + 3)^2 = 4^2 = 16 \quad \text{tandis que} \quad x^2 + 9 = 1 + 9 = 10.$$

Exercice 9 La troisième proposition est vraie. En effet, pour tout réel x , on a

$$(-x - 1)^2 = (-(x + 1))^2 = (x + 1)^2.$$

Exercice 10

- a. $(x + 6)^2 = x^2 + 12x + 36.$
- b. $(x - 10)^2 = x^2 - 20x + 100.$
- c. $(5x - 2)(5x + 2) = 25x^2 - 4.$
- d. $(2x + 5)^2 = 4x^2 + 20x + 25.$
- e. $(3x - 4)^2 = 9x^2 - 24x + 16.$
- f. $(7x + 3)(7x - 3) = 49x^2 - 9.$
- g. $\left(x + \frac{3}{2}\right)^2 = x^2 + 3x + \frac{9}{4}.$
- h. $\left(2x - \frac{1}{3}\right)^2 = 4x^2 - \frac{4}{3}x + \frac{1}{9}.$
- i. $(x - \sqrt{5})(x + \sqrt{5}) = x^2 - 5.$

Exercice 11

a.

$$\begin{aligned}
 2(x + 1)^2 - 1 &= 2(x^2 + 2x + 1) - 1 \\
 &= 2x^2 + 4x + 2 - 1 \\
 &= 2x^2 + 4x + 1.
 \end{aligned}$$

b.

$$\begin{aligned}
 5 - 4(x - 2)^2 &= 5 - 4(x^2 - 4x + 4) \\
 &= 5 - 4x^2 + 16x - 16 \\
 &= -4x^2 + 16x - 11.
 \end{aligned}$$

c.

$$\begin{aligned}
 -2(4x + 1)(4x - 1) &= -2(16x^2 - 1) \\
 &= -32x^2 + 2.
 \end{aligned}$$

Exercice 12

- a. $x^2 + 10x + 25 = (x + 5)^2.$
- b. $x^2 - 14x + 49 = (x - 7)^2.$
- c. $25x^2 - 16 = (5x + 4)(5x - 4).$
- d. $4x^2 + 12x + 9 = (2x + 3)^2.$
- e. $9x^2 - 30x + 25 = (3x - 5)^2.$
- f. $49x^2 - 81 = (7x + 9)(7x - 9).$
- g. $x^2 + \frac{8}{3}x + \frac{16}{9} = \left(x + \frac{4}{3}\right)^2.$

h. $4x^2 - \frac{25}{9} = \left(2x + \frac{5}{3}\right)\left(2x - \frac{5}{3}\right).$

i. $x^2 - 2 = (x + \sqrt{2})(x - \sqrt{2}).$

Exercice 13

a.

$$\begin{aligned} 3x^2 + 6x + 3 &= 3(x^2 + 2x + 1) \\ &= 3(x + 1)^2. \end{aligned}$$

b.

$$\begin{aligned} -5x^2 + 20x - 20 &= -5(x^2 - 4x + 4) \\ &= -5(x - 2)^2. \end{aligned}$$

c.

$$\begin{aligned} 32x^2 - 2 &= 2(16x^2 - 1) \\ &= 2(4x + 1)(4x - 1). \end{aligned}$$

Exercice 14

a. Pour tout réel x différent de -2 et de 1 , on a

$$\begin{aligned} \frac{3}{x+2} + \frac{5}{x-1} &= \frac{3(x-1)}{(x+2)(x-1)} + \frac{5(x+2)}{(x-1)(x+2)} \\ &= \frac{3(x-1) + 5(x+2)}{(x+2)(x-1)} \\ &= \frac{3x - 3 + 5x + 10}{(x+2)(x-1)} \\ &= \frac{8x + 7}{(x+2)(x-1)}. \end{aligned}$$

b. Pour tout réel x différent de 3 et de -1 , on a

$$\begin{aligned} \frac{x}{x-3} - \frac{2}{x+1} &= \frac{x(x+1)}{(x-3)(x+1)} - \frac{2(x-3)}{(x+1)(x-3)} \\ &= \frac{x(x+1) - 2(x-3)}{(x-3)(x+1)} \\ &= \frac{x^2 + x - 2x + 6}{(x-3)(x+1)} \\ &= \frac{x^2 - x + 6}{(x-3)(x+1)}. \end{aligned}$$

c. Pour tout réel x différent de 4 , on a

$$\begin{aligned} \frac{3}{x-4} + \frac{x-2}{2x-8} &= \frac{3 \times 2}{(x-4) \times 2} + \frac{x-2}{2x-8} \\ &= \frac{6}{2x-8} + \frac{x-2}{2x-8} \\ &= \frac{6+x-2}{2x-8} \\ &= \frac{x+4}{2x-8}. \end{aligned}$$

d. Pour tout réel x différent de 1 , on a

$$\begin{aligned} \frac{x+1}{2x-2} - \frac{x}{6x-6} &= \frac{(x+1) \times 3}{(2x-2) \times 3} - \frac{x}{6x-6} \\ &= \frac{3x+3}{6x-6} - \frac{x}{6x-6} \end{aligned}$$

$$\begin{aligned}
 &= \frac{3x + 3 - x}{6x - 6} \\
 &= \frac{2x + 3}{6x - 6}.
 \end{aligned}$$

e. Pour tout réel x différent de -4 , on a

$$\begin{aligned}
 2 + \frac{3}{x+4} &= \frac{2}{1} + \frac{3}{x+4} \\
 &= \frac{2 \times (x+4)}{1 \times (x+4)} + \frac{3}{x+4} \\
 &= \frac{2x+8}{x+4} + \frac{3}{x+4} \\
 &= \frac{2x+8+3}{x+4} \\
 &= \frac{2x+11}{x+4}.
 \end{aligned}$$

f. Pour tout réel x différent de 1 , on a

$$\begin{aligned}
 5 - \frac{2x}{x-1} &= \frac{5}{1} - \frac{2x}{x-1} \\
 &= \frac{5 \times (x-1)}{1 \times (x-1)} - \frac{2x}{x-1} \\
 &= \frac{5x-5}{x-1} - \frac{2x}{x-1} \\
 &= \frac{5x-5-2x}{x-1} \\
 &= \frac{3x-5}{x-1}.
 \end{aligned}$$

g. Pour tout réel x différent de 2 , on a

$$\begin{aligned}
 \frac{1}{2} + \frac{3}{x-2} &= \frac{1 \times (x-2)}{2 \times (x-2)} + \frac{3 \times 2}{(x-2) \times 2} \\
 &= \frac{x-2}{2x-4} + \frac{6}{2x-4} \\
 &= \frac{x-2+6}{2x-4} \\
 &= \frac{x+4}{2x-4}.
 \end{aligned}$$

h. Pour tout réel x différent de -3 et de 3 , on a

$$\begin{aligned}
 \frac{2}{x^2-9} - \frac{1}{x-3} &= \frac{2}{(x-3)(x+3)} - \frac{1}{x-3} \\
 &= \frac{2}{(x-3)(x+3)} - \frac{x+3}{(x-3)(x+3)} \\
 &= \frac{2-(x+3)}{(x-3)(x+3)} \\
 &= \frac{2-x-3}{(x-3)(x+3)} \\
 &= \frac{-x-1}{(x-3)(x+3)}.
 \end{aligned}$$

i. Pour tout réel x différent de 1 , de 0 et de -1 , on a

$$\begin{aligned}
\frac{1}{x-1} + \frac{1}{x} + \frac{1}{x+1} &= \frac{x(x+1)}{x(x-1)(x+1)} + \frac{(x-1)(x+1)}{x(x-1)(x+1)} + \frac{x(x-1)}{x(x-1)(x+1)} \\
&= \frac{x(x+1) + (x-1)(x+1) + x(x-1)}{x(x-1)(x+1)} \\
&= \frac{x^2 + x + (x^2 - 1) + (x^2 - x)}{x(x-1)(x+1)} \\
&= \frac{3x^2 - 1}{x(x-1)(x+1)}.
\end{aligned}$$

Exercice 15**a.**

$$\begin{aligned}
A &= (x+7)^2 + 3(x+7) \\
&= x^2 + 14x + 49 + 3x + 21 \\
&= x^2 + 17x + 70.
\end{aligned}$$

b.

$$\begin{aligned}
A &= (x+7)^2 + 3(x+7) \\
&= (x+7)[(x+7) + 3] \\
&= (x+7)(x+10).
\end{aligned}$$

c. Pour $x = -7$, on a

$$A = (-7+7)(7+10) = 0 \times 17 = 0.$$

Exercice 16**a.**

$$\begin{aligned}
A &= 25x^2 + 10x + 1 - (3-2x)(5x+1) \\
&= 25x^2 + 10x + 1 - (15x + 3 - 10x^2 - 2x) \\
&= 25x^2 + 10x + 1 - (-10x^2 + 13x + 3) \\
&= 25x^2 + 10x + 1 + 10x^2 - 13x - 3 \\
&= 35x^2 - 3x - 2.
\end{aligned}$$

b. On a

$$25x^2 + 10x + 1 = (5x+1)^2,$$

puis

$$\begin{aligned}
A &= (5x+1)^2 - (3-2x)(5x+1) \\
&= (5x+1)(5x+1) - (3-2x)(5x+1) \\
&= (5x+1)[(5x+1) - (3-2x)] \\
&= (5x+1)[5x+1-3+2x] \\
&= (5x+1)(7x-2).
\end{aligned}$$

c. Pour $x = -\frac{1}{5}$, on a

$$A = \left(5 \times \left(-\frac{1}{5}\right) + 1\right) \left(7 \times \left(-\frac{1}{5}\right) - 2\right) = (-1+1) \left(7 \times \left(-\frac{1}{5}\right) - 2\right) = 0 \times \left(7 \times \left(-\frac{1}{5}\right) - 2\right) = 0.$$

Exercice 17**a.**

$$\begin{aligned}
A &= 49 - (2x+1)^2 \\
&= 49 - (4x^2 + 4x + 1)
\end{aligned}$$

$$\begin{aligned} &= 49 - 4x^2 - 4x - 1 \\ &= -4x^2 - 4x + 48. \end{aligned}$$

b.

$$\begin{aligned} A &= 49 - (2x + 1)^2 \\ &= 7^2 - (2x + 1)^2 \\ &= [7 + (2x + 1)][7 - (2x + 1)] \\ &= [7 + 2x + 1][7 - 2x - 1] \\ &= (2x + 8)(-2x + 6) \\ &= 2(x + 4) \times 2(-x + 3) \\ &= 4(x + 4)(3 - x). \end{aligned}$$

c. Pour $x = 3$, on a

$$A = 4(3 + 4)(3 - 3) = 4 \times 7 \times 0 = 0.$$